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Dynamical symmetry breaking in Gauge-Higgs unification of 5D $\mathcal{N}=1$ SUSY theory

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Abstract: We study the dynamical symmetry breaking in the gauge-Higgs unification of the 5D $\mathcal{N}=1$ SUSY theory, compactified on an orbifold, $S^1/\mathbb{Z}_2$. This theory identifies Wilson line degrees of freedoms as "Higgs doublets". We consider $\mathrm{SU}(3)_c \times \mathrm{SU}(3)_W$ and SU(6) models, in which the gauge symmetries are reduced to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)$, respectively, through the orbifolding boundary conditions. Quarks and leptons are bulk fields, so that Yukawa interactions can be derived from the 5D gauge interactions. We estimate the one loop effective potential of "Higgs doublets", and analyze the vacuum structures in these two models. We find that the effects of bulk quarks and leptons destabilize the suitable electro-weak vacuum. We show that the introduction of suitable numbers of extra bulk fields possessing the suitable representations can realize the appropriate electro-weak symmetry breaking.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, Spontaneous Symmetry Breaking, Compactification and String Models.

Contents

1. Introduction

Recently, there are much progress in the higher dimensional gauge theories. One of the most fascinating motivations of considering the higher dimensional gauge theory is that gauge and Higgs fields can be unified [1]–[11]. We call this scenario as the gauge-Higgs unification. The higher dimensional components of gauge fields become scalar fields below the compactification scale. These scalar fields are identified with the "Higgs fields" in the gauge-Higgs unified models. The "Higgs fields" have only finite masses of order the compactification scale, since the masses of "Higgs fields" are forbidden by the higher dimensional gauge invariance. The "adjoint Higgs fields" can be induced through the S^1 compactification in 5D theory, while the "Higgs doublet fields" can be induced through the orbifold compactifications. In order to obtain the "Higgs doublets" from the gauge fields in higher dimensions, the gauge group must be lager than the standard model (SM) gauge group. The gauge symmetries are reduced by the orbifolding boundary conditions of extra dimensions. The identification of "Higgs fields" as a part of the gauge supermultiplet has been considered in 5D $\mathcal{N}=1$ supersymmetric (SUSY) gauge theory whose 5th coordinate is compactified on $S^1/\mathbb{Z}_2$ orbifold [4]–[7] and [9, 11]. Also in 6D $\mathcal{N}=2$ SUSY gauge theory,[1] the gauge-Higgs unification has been considered on $T^2/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orbifold [5].

This paper considers the former scenario, the 5D $\mathcal{N}=1$ SUSY theories compactified on an orbifold, $S^1/\mathbb{Z}_2$, in which the Wilson line degrees of freedoms (d.o.f.) can be identified as "Higgs doublets". We consider $\mathrm{SU}(3)_c \times \mathrm{SU}(3)_W$ and $\mathrm{SU}(6)$ models, where the gauge symmetries are reduced to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)$, respectively, through the orbifolding boundary conditions. The case that quarks and leptons are localized on the 4D walls has been studied in ref. [12]. This paper is concentrating on the case that quarks and leptons are bulk fields. In this case, Yukawa interactions can be derived from the 5D gauge interactions [6]. We calculate the one loop

[1] The 6D $\mathcal{N}=2$ SUSY gauge theory corresponds to the 4D $\mathcal{N}=4$ SUSY gauge theory.

effective potential [12]–[15] of "Higgs doublets", and analyze the vacuum structure of the models. We find that the effects of bulk quarks and leptons destabilize the suitable electro-weak vacuum. We show that the introduction of suitable numbers of extra bulk fields possessing the suitable representations makes two appropriate scenarios be possible. One is the situation that the one loop effective potential chooses symmetric vacuum at the high energy (compactification) scale, and the electro-weak symmetry breaking is realized by other effects [11] in the low energy. The other is the situation that the one loop effective potential chooses the suitable electro-weak vacuum in a few TeV compactification scale.[2] Here the masses of "Higgs doublets" become $\mathcal{O}(100)$ GeV.

2. Gauge-Higgs unification on $S^1/\mathbb{Z}_2$

At first let us show the notation of the 5D $\mathcal{N}=1$ SUSY gauge theory, which corresponds to the 4D $\mathcal{N}=2$ SUSY gauge theory [16]. The gauge supermultiplet, (V,Σ), of the $\mathcal{N}=2$ SUSY gauge theory is written as

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu + i\bar{\theta}^2\theta\lambda - i\theta^2\bar{\theta}\bar{\lambda} + \frac{1}{2}\bar{\theta}^2\theta^2 D\,, \tag{2.1}$$

$$\Sigma = \frac{1}{\sqrt{2}}(\sigma + iA_5) + \sqrt{2}\theta\lambda' + \theta^2 F\,. \tag{2.2}$$

A_5 is the 5th component of the 5D gauge field. In the non-abelian gauge theory, the gauge transformation is given by $e^V \to h^{-1}e^V\bar{h}^{-1}$ and $\Sigma \to h^{-1}(\Sigma + \sqrt{2}\partial_y)h$, where we denote $h \equiv e^{-\Lambda}$, $\bar{h} \equiv e^{-\bar{\Lambda}}$ and $V \equiv V^aT^a$, $\Sigma \equiv \Sigma^aT^a$, $\Lambda \equiv \Lambda^aT^a$. Then the action is given by

$$\begin{aligned} S_{5D} = \int d^4x dy \Bigg[& \frac{1}{4kg^2}\mathrm{Tr}\left\{\int d^2\theta W^\alpha W_\alpha + h.c.\right\} + \\ & + \int d^4\theta \frac{1}{kg^2}\mathrm{Tr}\left((\sqrt{2}\bar{\partial}_y + \bar{\Sigma})e^{-V}(-\sqrt{2}\partial_y + \Sigma)e^V + \bar{\partial}e^{-V}\partial e^V\right)\Bigg], \end{aligned} \tag{2.3}$$

where $\mathrm{Tr}(T^aT^b) = k\delta_{ab}$.

As for the hypermultiplet, (H, H^c), they transform $H \to hH$ and $H^c \to h^{c-1}H^c$ under the gauge transformation, where $h = e^{-\Lambda^aT^a}$ and $h^c = (h^{-1})^T = (e^{\Lambda^aT^a})^T$. The action of them is given by

$$\begin{aligned} S^H_{5D} = \int d^4x dy \Bigg[& \int d^4\theta (H^c e^V \bar{H}^c + \bar{H}e^{-V}H) + \\ & + \left[\int d^2\theta \left(H^c\left(m + \left(\partial_y - \frac{1}{\sqrt{2}}\Sigma\right)\right)H\right) + \mathrm{h.c.}\right]\Bigg]. \end{aligned} \tag{2.4}$$

This means that Σ must change the sign under the $\mathbb{Z}_2$ projection, $P : y \to -y$, and bulk constant mass m is forbidden.[3] The field H^c is so-called mirror field, which should have the opposite parity of P compared to H, since it is the right-handed field. Thus, the parity

[2] We should assume the baryon number symmetry to avoid rapid proton decay.

[3] When the y dependent mass $m(-y) = -m(y)$ is introduced, it makes the zero mode wave-function in the bulk be localized at $y = 0$ [17].

operator P acts on fields as

$$V(y) = PV(-y)P^{-1}\,, \qquad \Sigma(y) = -P\Sigma(-y)P^{-1}\,, \tag{2.5}$$

$$H(y) = \eta T[P]H(-y)\,, \qquad H^c(y) = -\eta T[P]H^c(-y)\,. \tag{2.6}$$

$T[P]$ represents an appropriate representation matrix, for example, when H is the fundamental or adjoint representation, $T[P]H$ means PH or $PHP^\dagger$, respectively. The parameter, η is like a *intrinsic* parity eigenvalue, which takes ± 1. As for the reflection around $y = \pi R$ denoted as P', which is reproduced by the product of transformations, $y \to -y$ and $y \to y + 2\pi R$, the bulk fields transform as

$$V(\pi R - y) = P'V(\pi R + y)P'^{-1}\,, \qquad \Sigma(\pi R - y) = P'\Sigma(\pi R + y)P'^{-1}, \tag{2.7}$$

$$H(\pi R - y) = \eta' T[P']H(\pi R + y)\,, \qquad H^c(\pi R - y) = \eta' T[P']H^c(\pi R + y)\,, \tag{2.8}$$

where $\eta' = \pm$.

We will consider nontrivial P and P' in the gauge group base in order to regard the zero mode components of Σ chiral superfield as the "Higgs doublet". In this case, the F-term interaction in eq. (2.4)

$$W_Y \supset H^c \Sigma H\,, \tag{2.9}$$

which is invariant under the $\mathbb{Z}_2$ projections, can be regarded as the "Yukawa interactions". The interaction in eq. (2.9) connects the chiral and mirror fields through the chirality flip, which seems to be really Yukawa interaction in the 4D theory. This theory proposes that the origin of "Yukawa interactions" exist in the 5D gauge interactions.

3. $\mathrm{SU}(3)_c \times \mathrm{SU}(3)_W$ model

Now let us study the $\mathrm{SU}(3)_c \times \mathrm{SU}(3)_W$ model, where the Higgs doublets can be identified as the zero mode components of Σ [4]–[6]. We consider the case that quarks and leptons are introduced in the bulk to produce Yukawa interactions as eq. (2.9) [6, 7, 9]. We analyze the vacuum structure of this model.

This model takes parities as

$$P = P' = \mathrm{diag}(1, -1, -1), \tag{3.1}$$

in the base of $\mathrm{SU}(3)_W$,[4] which divide V and Σ as

$$V = \left(\begin{array}{c|cc} (+,+) & (-,-) & (-,-) \\ \hline (-,-) & (+,+) & (+,+) \\ (-,-) & (+,+) & (+,+) \end{array}\right), \tag{3.2}$$

$$\Sigma = \left(\begin{array}{c|cc} (-,-) & (+,+) & (+,+) \\ \hline (+,+) & (-,-) & (-,-) \\ (+,+) & (-,-) & (-,-) \end{array}\right). \tag{3.3}$$

[4] We take $P = P' = I$ for $\mathrm{SU}(3)_c$.

This suggests that $\mathrm{SU}(3)_W$ is broken down to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, and there appear two "Higgs doublet" superfields as the zero modes of Σ.[5]

The scalar component of Σ can take the vacuum expectation value (VEV) written as $\frac{1}{gR}\sum_a a_a \frac{\lambda_a}{2}$. We can always take VEV as

$$\langle\Sigma\rangle = \frac{1}{2gR}\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & 0 \\ a & 0 & 0 \end{pmatrix} \tag{3.4}$$

by using the residual $\mathrm{SU}(2)\times\mathrm{U}(1)$ *global* symmetry.[6] We adopt Scherk-Schwarz (SS) SUSY breaking [18]–[21]. The effective potential induced from the gauge sector is given by [12, 13]

$$V_{\mathrm{eff}}^{\mathrm{gauge}} = -2C\sum_{n=1}^{\infty}\frac{1}{n^5}(1-\cos(2\pi n\beta))[\cos(2\pi na)+2\cos(\pi na)]\,, \tag{3.5}$$

where $C \equiv 3/(64\pi^7R^5)$. The β parameterizes SS SUSY breaking, for which we take $\beta/R = \mathcal{O}(100)\,\mathrm{GeV}$, since the soft mass is given by $\mathcal{O}(\beta/R)$ [14].

Next, let us calculate the effective potential induced from the quarks and leptons in the bulk. As in ref.[6], we introduce **3**, **6**, **10**, and **8** representation hypermultiplets of bulk quarks and leptons in order to reproduce the Yukawa interactions of up-, down-, charged lepton-, and neutrino-sectors, respectively. They all possess $\eta\eta' = +$ in eqs. (2.6) and (2.8). Their contributions to the effective potential are given by

$$\begin{aligned} V_{\mathrm{eff}}^{q/l} &= 2N_gC\sum_{n=1}^{\infty}\frac{1}{n^5}(1-\cos(2\pi n\beta))\times[3f_u(a)+3f_d(a)+f_e(a)+f_\nu(a)]\,, && (3.6)\\ f_u(a) &= \cos(\pi na)\,, && (3.7)\\ f_d(a) &= \cos(2\pi na)+\cos(\pi na)\,, && (3.8)\\ f_e(a) &= \cos(3\pi na)+\cos(2\pi na)+2\cos(\pi na)\,, && (3.9)\\ f_\nu(a) &= \cos(2\pi na)+2\cos(\pi na)\,, && (3.10)\end{aligned}$$

where N_g is the generation number of quarks and leptons in the bulk, and $f_u(a)$, $f_d(a)$, $f_e(a)$, and $f_\nu(a)$, are contribution from up-, down-, charged lepton-, and neutrino-sectors, respectively. The coefficients 3 in front of $f_u(a)$ and $f_d(a)$ denote the color factors. The contributions from the fundamental, **3**, the symmetric tensor, **6**, and the adjoint, **8**, representations are shown in the general formula in ref. [13]. The remaining contribution from **10** representation can be calculated by use of the calculation method in ref. [13] as follows.

[5]There appears one "Higgs doublet" as a zero mode of A_5 in a non-SUSY theory.

[6]Since this is the D-flat direction, there does not appear the tree-level quartic couplings in the effective potential. We can show that the vacuum exists on the D-flat direction under the Scherk-Schwarz SUSY breaking. Some low energy contributions might make the vacuum off the direction, but we assume it is not so large. Anyway, our scenario shows $\tan\beta \simeq 1$.

JHEP04(2004)016

The VEV in eq. (3.4) is proportional to one generator of $\mathrm{SU}(2)_{13}$ that operates on the 2×2 submatrix of $(1,1)$, $(1,3)$, $(3,1)$ and $(3,3)$ components. The $\mathbf{10}$ is decomposed as $\mathbf{4}+\mathbf{3}+\mathbf{2}+\mathbf{1}$ under the base of $\mathrm{SU}(2)_{13}$. The U(1) charge for the VEV direction, eq. (3.4), is given by

$$(\underbrace{+3/2,+1/2,-1/2,-3/2}_{\mathbf{4}},\underbrace{+1,-1,0}_{\mathbf{3}},\underbrace{+1/2,-1/2}_{\mathbf{2}},\underbrace{0}_{\mathbf{1}})\,. \tag{3.11}$$

This means that the eigenvalues of $D_y(A_5)^2$ for a $\mathbf{10}$ representation field B are

$$2\times\frac{n^2}{R^2}\,,\qquad \frac{(n\pm 3a/2)^2}{R^2}\,,\qquad \frac{(n\pm a)^2}{R^2}\,,\qquad 2\times\frac{(n\pm a/2)^2}{R^2}\,. \tag{3.12}$$

Here eigenfunctions of B are expanded as $B \propto \cos\frac{ny}{R}, \sin\frac{ny}{R}$, since (P,P') of components of B are either $(+,+)$ or $(-,-)$. Therefore, the contribution from the $\mathbf{10}$ representation to the effective potential is

$$\begin{aligned}
&\frac{i}{2}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{1}{2\pi R}\sum_{n=-\infty}^{\infty}\left[\ln\left(-p^2+\left(\frac{n}{R}\right)^2\right)+\ln\left(-p^2+\left(\frac{n-3a/2}{R}\right)^2\right)+\right.\\
&\qquad\qquad\left.+\ln\left(-p^2+\left(\frac{n-a}{R}\right)^2\right)+2\ln\left(-p^2+\left(\frac{n-a/2}{R}\right)^2\right)\right].\\
&=\frac{C}{2}\sum_{n=1}^{\infty}\frac{1}{n^5}\left[\cos(3\pi na)+\cos(2\pi na)+2\cos(\pi na)\right],
\end{aligned} \tag{3.13}$$

up to a-independent terms, for one degree of freedom of fermion. This is the derivation of eq. (3.9).

Then, the total effective potential is given by

$$\begin{aligned}
V_{\mathrm{eff}} &= V_{\mathrm{eff}}^{\mathrm{gauge}}+V_{\mathrm{eff}}^{q/l}\\
&=2C\sum_{n=1}^{\infty}\frac{1}{n^5}(1-\cos(2\pi n\beta))\times\\
&\quad\times\left[N_g\cos(3\pi na)+(5N_g-1)\cos(2\pi na)+(10N_g-2)\cos(\pi na)\right].
\end{aligned} \tag{3.14}$$

Seeing the 1st derivative of V_{eff}, each term of $\partial V_{\mathrm{eff}}/\partial a$ has a factor $\sin(\pi na)$, which means that the stationary points exist at least at $a=0$ and $a=1$.[7] The difference of the heights between two points is given by

$$V_{\mathrm{eff}}(a=0)-V_{\mathrm{eff}}(a=1)=4(11N_g-2)C\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}(1-\cos(2\pi(2n-1)\beta))\,. \tag{3.15}$$

This means that the height of $a=0$ point is higher than that of $a=1$ even when there are one generation quarks and leptons in the bulk. The vacuum at $a=1$ point induces the

[7] The potential has the symmetry $V_{\mathrm{eff}}(-a)=V_{\mathrm{eff}}(a)$, so that we should only check the region of $0\le a\le 1$.

Wilson loop,

$$
\begin{aligned}
W_C &= \exp\left(ig \int_0^{2\pi R} dy \, \langle \Sigma \rangle \right) \\
&= \exp\left(ig 2\pi R \frac{1}{2gR} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix}, \qquad (3.16)
\end{aligned}
$$

which suggests $SU(2)_L \times U(1)_Y$ is broken down to $U(1)_{EM} \times U(1)$ at the energy scale of $\mathcal{O}(R^{-1})$. So this case can not reproduce the correct weak scale VEV, since the compactification scale should be higher than the weak scale. Notice here that it does not mean the model in ref. [6] is incorrect. There the suitable background VEV is introduced in order to make bulk fields localized at $a = 0$ and $a = 1$ and to achieve the fermion mass hierarchy. The authors also introduce the wall-bulk mixing mass terms, which make unwanted zero modes, such as triplet of $SU(3)_W$ in **6**, be heavy. Our calculations do not include these effects, so that our analyses of vacuum above are not for the model in ref. [6]. In this paper, we take the standing point that the fermion mass hierarchy should be reproduced by another mechanism (as that shown in ref. [7], where localization of bulk fields is not needed), and neglect the effects of wall-bulk mixing mass terms, which is justified when the compactification scale is higher than these mass terms.[8] In this situation, we can conclude that the vacuum does not realize the suitable electro-weak symmetry breaking. So this model should be modified as realizing the suitable vacuum for the realistic gauge-Higgs unification scenario.

We introduce extra fields in the bulk for the suitable electro-weak symmetry breaking. We should impose a discrete symmetry in order to avoid unwanted Yukawa interactions between ordinary particles and extra fields. We consider the following two possibilities;

1. The one loop effective potential chooses $a = 0$ vacuum at the high energy (compactification) scale, and the electro-weak symmetry breaking is realized by other effects, such as [11], in the low energy.

2. In the TeV scale compactification, the suitable electro-weak symmetry breaking is obtained through the one loop effective potential.

We realize these two situations by introducing extra bulk fields of $N_f^{(\pm)}$ and $N_a^{(\pm)}$ species of hypermultiplets of fundamental and adjoint representations, respectively [12], where index $(\pm)$ denotes the sign of $\eta\eta'$. We should take even number of $N_f^{(\pm)}$ to avoid the gauge anomaly. The extra matter contributions for the effective potential are given by [12, 13]

$$
\begin{aligned}
V_{\text{eff}}^{extra-m} &= 2C \sum_{n=1}^{\infty} \frac{1}{n^5} (1 - \cos(2\pi n \beta)) \times \\
&\times \Big[N_a^{(+)} \cos(2\pi n a) + N_a^{(-)} \cos(\pi n(2a-1)) + \left(2N_a^{(+)} + N_f^{(+)}\right) \cos(\pi n a) + \\
&\quad + \left(2N_a^{(-)} + N_f^{(-)}\right) \cos(\pi n(a-1)) \Big]. \qquad (3.17)
\end{aligned}
$$

[8] The effect to the effective potential from wall-bulk mixing mass terms can be vanished due to the parity assignment. We thank N. Okada for pointing out this.

The total effective potential is $V_{\rm eff} = V_{\rm eff}^{gauge} + V_{\rm eff}^{q/l} + V_{\rm eff}^{extra-m}$. The vacuum of this effective potential has the

$$\begin{aligned}
&(1):\ \text{unbroken phase } (a=0)\,,\\
&(2):\ \text{broken phase I } (a=1)\,,\\
&(3):\ \text{broken phase II } (a\neq 0,1)\,,
\end{aligned}\tag{3.18}$$

in which remaining gauge symmetries are (1): $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, (2): $\mathrm{U}(1)_{\rm EM} \times \mathrm{U}(1)$, and (3): $\mathrm{U}(1)_{\rm EM}$, respectively. We can see 1st derivatives of $V_{\rm eff}^{extra-m}$ at $a = 0$ and $a = 1$ vanish, thus, that of the total effective potential also dose. The stability of each stationary point is determined by the 2nd derivative of the effective potential evaluated at the point as examined in ref. [22]. By using the approximation formula for a small (positive) ξ,

$$\sum_{n=1}^{\infty} \frac{\cos(n\xi)}{n^3} \simeq \zeta_R(3) + \frac{\xi^2}{2}\ln\xi - \frac{3}{4}\xi^2\,, \tag{3.19}$$

$$\sum_{n=1}^{\infty} \frac{\cos(n\xi)}{n^3}(-1)^n \simeq -\frac{3}{4}\zeta_R(3) + \frac{\xi^2}{2}\ln 2\,, \tag{3.20}$$

where $\zeta_R(z)$ is the Riemann's zeta function, the 2nd derivatives are approximated as

$$\begin{aligned}
\left.\frac{\partial^2 V_{\rm eff}}{\partial a^2}\right|_{a=0} &= -2C\pi^2 \sum_{n=1}^{\infty} \frac{1}{n^3}\left(1-\cos(2\pi n\beta)\right) \times \\
&\quad \times \left[9N_g + 4\left(5N_g - 1 + N_a^{(+)}\right) + 4N_a^{(-)}(-1)^n + \right.\\
&\quad \left. + \left(10N_g - 2 + 2N_a^{(+)} + N_f^{(+)}\right) + \left(2N_a^{(-)} + N_f^{(-)}\right)(-1)^n \right]\\
&\simeq 2C\pi^2(2\pi\beta)^2 \left[\left(39N_g - 6 + 6N_a^{(+)} + N_f^{(+)}\right)\left(-\frac{1}{2}\ln(2\pi\beta) + \frac{3}{4}\right) + \right.\\
&\quad \left. + \left(-6N_a^{(-)} - N_f^{(-)}\right)\frac{1}{2}\ln 2 \right],
\end{aligned}\tag{3.21}$$

$$\begin{aligned}
\left.\frac{\partial^2 V_{\rm eff}}{\partial a^2}\right|_{a=1} &= -2C\pi^2 \sum_{n=1}^{\infty} \frac{1}{n^3}\left(1-\cos(2\pi n\beta)\right) \times \\
&\quad \times \left[9N_g(-1)^n + 4\left(5N_g - 1 + N_a^{(+)}\right) + 4N_a^{(-)}(-1)^n + \right.\\
&\quad \left. + \left(10N_g - 2 + 2N_a^{(+)} + N_f^{(+)}\right)(-1)^n + \left(2N_a^{(-)} + N_f^{(-)}\right) \right]\\
&\simeq 2C\pi^2(2\pi\beta)^2 \left[\left(20N_g - 4 + 4N_a^{(+)} + 2N_a^{(-)} + N_f^{(-)}\right)\left(-\frac{1}{2}\ln(2\pi\beta) + \frac{3}{4}\right) + \right.\\
&\quad \left. + \left(-19N_g + 2 - 2N_a^{(+)} - 4N_a^{(-)} - N_f^{(+)}\right)\frac{1}{2}\ln 2 \right].
\end{aligned}\tag{3.22}$$

The point $a = 0\,(a = 1)$ is stable when $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=0} > 0$ ($\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=1} > 0$). Whether these points become the true vacuum (the global minimum) or not, more detailed analyses are needed since there is the possibility to find local minimums at other points, in general. However, the numerical analyses show the following results, at least, when the numbers of bulk fields are not extremely large.

- When $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=0} > 0$ and $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=1} < 0$, the stationary point $a = 0$ becomes a global minimum, and the unbroken phase is realized.
- When $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=0} < 0$ and $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=1} > 0$, the stationary point $a = 1$ becomes a global minimum, and the broken phase I is realized.
- When $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=0} > 0$ and $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=1} > 0$, either two vacua are degenerated as a global minimum or one of the points of $a = 0$ and $a = 1$ becomes a global minimum, depending on bulk fields contents.

They can be understood by the following discussion. Since $n = 1$ (of the summation of n) and $n = 2$ (when contributions of $n = 1$ are canceled between $\eta\eta' = +$ part and $\eta\eta' = -$ part) have dominant contributions to the form of the effective potential, the effective potential is approximated as a sum of $\pm\cos(\pi a)$, $\pm\cos(2\pi a)$, $\cos(3\pi a)$ and $\cos(4\pi a)$. When, for example, $\left.\frac{\partial^2 V_{\rm eff}}{\partial a^2}\right|_{a=0} > 0$, $-\cos(\pi a)$ and/or $-\cos(2\pi a)$ must dominate the effective potential. This means that there is no global minimum between $a = 0$ and $a = 1$, when $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=0}$ and/or $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=1}$ are positive. It is justified as long as there are no higher representation fields in the bulk that induce the other terms (e.g. $-\cos(3\pi a)$).

The remaining case is as follows.

- When $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=0} < 0$ and $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=1} < 0$, there is a global minimum at $a \neq 0, 1$, and the broken phase II is realized.

It is worth noting that the phase structure is completely determined by the bulk fields contents in this situation.

Above discussion suggests that we can obtain a vanishing VEV, $a = 0$, as the global minimum when $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=0} > 0$ and $V_{\rm eff}|_{a=0} < V_{\rm eff}|_{a=1}$. And, we can obtain a small VEV, $(0 <)a \ll 1$, when we choose a point near the region corresponding to the unbroken phase in the parameter space. Namely, when $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=0} \lesssim 0$ and $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=1} < 0$, a small VEV will be obtained.[9]

Now let us consider the situation all three generation quarks and leptons exist in the bulk. Equation (3.21) suggests that at least $\mathcal{O}(10)$ numbers of $N_a^{(-)}$ and/or $N_f^{(-)}$ are required for realizing vacuum at $a = 0$ and $a \ll 1$, since $(-\frac{1}{2}\ln(2\pi\beta) + \frac{3}{4})/(\frac{1}{2}\ln 2) \sim 3$ for $\beta = 0.1$. The former situation, $a = 0$, can be realized when the number of extra bulk fields are $N_a^{(+)} = 0$, $N_f^{(+)} = 0$, $N_a^{(-)} = 50$, and $N_f^{(-)} = 50$, for examples. For realizing the latter

[9]In ref. [12] we have obtained the bulk field content by finding the case that induce large coefficients of $-\cos(\pi n a)$ and/or $\cos(\pi n(a-1))$, and small (but non-zero) coefficients of $\cos(2\pi n a)$ and $-\cos(2\pi n(a-1/2))$. This approach of finding the suitable bulk field content inducing a ($a \ll 1$) is generalized by the approach of finding the cases of $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=0} \lesssim 0$ and $\frac{\partial^2 V_{\rm eff}}{\partial a^2}|_{a=1} < 0$.

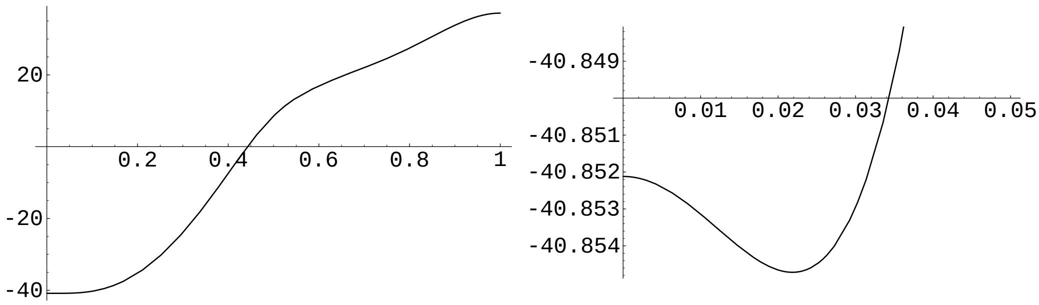

Figure 1: The effective potential in the case of $N_a^{(+)} = N_f^{(+)} = 0$, $N_a^{(-)} = 45$, $N_f^{(-)} = 40$ with $\beta = 0.1$ and $N_g = 3$. The unit is $C = 3/64\pi^7 R^5$. The horizontal line shows $0 \le a \le 1$ and $0 \le a \le 0.05$.

situation, we assume that the compactification scale is of $\mathcal{O}(1)$ TeV[10] and wall-bulk mass scale is of $\mathcal{O}(100)$ GeV. The suitable electro-weak symmetry breaking is realized when the global minimum exists at $a \ll 1$, and one example of the parameter set for realizing it is $N_a^{(+)} = 0$, $N_f^{(+)} = 0$, $N_a^{(-)} = 45$, and $N_f^{(-)} = 40$. Figure 1 shows the $V_{\rm eff}$ in the region of $0 \le a \le 1$ and $0 \le a \le 0.05$. The minimum exists at $a = 0.022$, which is around the suitable magnitude of the weak scale in the case of TeV scale compactification. The kinetic term of the "Higgs field" can be reproduced through the effects of wall-localized kinetic terms[6]. Setting $\langle A_5 \rangle / \sqrt{2\pi R} = a/g_4 R \sim 246\,\text{GeV}$, the mass squared of the "Higgs field" is given by

$$m_{A_5}^2 = (gR)^2 \left. \frac{\partial^2 V_{\rm eff}}{\partial a^2} \right|_{a=0.022} = \frac{3g_4^2}{32\pi^4 R^2} \left. \frac{\partial^2 (V_{\rm eff}/(C\pi^2))}{\partial a^2} \right|_{a=0.022} . \tag{3.23}$$

Numerical analysis shows

$$m_{A_5}^2 \sim \left(\frac{0.062 g_4}{R} \right)^2 \sim (700 g_4^2\,\text{GeV})^2 , \tag{3.24}$$

where the 4D gauge coupling constant $g_4 \equiv g/\sqrt{2\pi R}$ is expected to be of $\mathcal{O}(1)$. We take $\beta = 0.1$, since the soft mass is given by β/R. We should notice that the suitable global minimum can be realized by introducing less numbers of extra bulk fields, when only the 2nd and 3rd generations, or only the 3rd generation quarks and leptons spread in the bulk.

4. SU(6) model

Next, we study the vacuum structure of the SU(6) model, in which the Higgs doublets can be identified as the zero mode components of Σ [5, 6]. We take

$$P = \text{diag}(1, 1, 1, 1, -1, -1) , \tag{4.1}$$

$$P' = \text{diag}(1, -1, -1, -1, -1, -1) , \tag{4.2}$$

[10] In this paper we assume that the effect of wall-localized kinetic terms to the effective potential [8] is negligible. We need to check whether the suitable value of $\sin^2\theta_W$ and the gauge coupling unification are realized by the effects of wall-localized kinetic terms without affecting the effective potential. For the possibilities, the power law unification[23] or the accelerated unification[24] might be useful.

in the base of SU(6), which divide V and Σ as

$$V = \left(\begin{array}{c|ccc|cc} (+,+) & (+,-) & (+,-) & (+,-) & (-,-) & (-,-) \\ \hline (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,+) \\ (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,+) \\ (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,+) \\ \hline (-,-) & (-,+) & (-,+) & (-,+) & (+,+) & (+,+) \\ (-,-) & (-,+) & (-,+) & (-,+) & (+,+) & (+,+) \end{array}\right), \tag{4.3}$$

$$\Sigma = \left(\begin{array}{c|ccc|cc} (-,-) & (-,+) & (-,+) & (-,+) & (+,+) & (+,+) \\ \hline (-,+) & (-,-) & (-,-) & (-,-) & (+,-) & (+,-) \\ (-,+) & (-,-) & (-,-) & (-,-) & (+,-) & (+,-) \\ (-,+) & (-,-) & (-,-) & (-,-) & (+,-) & (+,-) \\ \hline (+,+) & (+,-) & (+,-) & (+,-) & (-,-) & (-,-) \\ (+,+) & (+,-) & (+,-) & (+,-) & (-,-) & (-,-) \end{array}\right). \tag{4.4}$$

They suggest that P and P' make SU(6) break to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)$. Also, there appears two "Higgs doublet" superfields as the zero modes of Σ. The VEV of Σ is written as

$$\frac{1}{2gR}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \equiv \frac{1}{gR} a \frac{\lambda}{2} \tag{4.5}$$

by using the residual $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)$ *global* symmetry. The gauge contribution of the effective potential in the SU(6) model is given by [12, 13]

$$\begin{aligned} V_{\text{eff}}^{gauge} &= -2C \sum_{n=1}^{\infty} \frac{1}{n^5}(1 - \cos(2\pi n\beta)) \times \\ &\quad \times \left[6\cos(\pi n(a-1)) + 2\cos(\pi n a) + \cos(2\pi n a)\right]. \end{aligned} \tag{4.6}$$

As for the quarks and leptons, we introduce **20**, **15**, **15**, and **6** representation bulk hypermultiplets in order to reproduce the Yukawa interactions of up-, down-, charged lepton-, and neutrino-sectors, respectively [6]. The up- and down-sector fields, **20** and **15**, possess $\eta\eta' = -$, while the charged lepton- and neutrino-sector fields, **15** and **6**, possess $\eta\eta' = +$ [6]. Their contributions to the effective potential are given by

$$V_{\text{eff}}^{q/l} = 2N_g C \sum_{n=1}^{\infty} \frac{1}{n^5}(1 - \cos(2\pi n\beta)) \times \left[g_u(a) + g_d(a) + g_e(a) + g_\nu(a)\right], \tag{4.7}$$

$$g_u(a) = 3\cos(\pi n a) + 3\cos(\pi n(a-1)), \tag{4.8}$$

$$g_d(a) = 3\cos(\pi n a) + 2\cos(\pi n(a-1)), \tag{4.9}$$

$$g_e(a) = 2\cos(\pi n a) + 3\cos(\pi n(a-1)), \tag{4.10}$$

$$g_\nu(a) = \cos(\pi n a), \tag{4.11}$$

where N_g is the generation number, and $g_u(a)$, $g_d(a)$, $g_e(a)$, and $g_\nu(a)$, are contribution from up-, down-, charged lepton-, and neutrino-sectors, respectively. The contributions from the fundamental, **6**, and anti-symmetric tensor, **15**, representations are shown in the general formula in ref. [13]. The remaining contributions from **20** can be also calculated by using the same calculation method in ref. [13]. As suggested there, the product of parities (eq. (4.1) and eq. (4.2)),

$$PP' = \mathrm{diag}(1,-1,-1,-1,1,1)\,, \tag{4.12}$$

by which the gauge symmetry is reduced as $\mathrm{SU}(6) \to \mathrm{SU}(3)_c \times \mathrm{SU}(3)_L \times \mathrm{U}(1)$, plays an important role. In this case, the generator λ in eq. (4.5) is one of the generators of $\mathrm{SU}(3)_L$. The **20** is decomposed as

$$\mathbf{35} \to (\mathbf{3},\bar{\mathbf{3}},+) + (\mathbf{1},\mathbf{1},+) + (\bar{\mathbf{3}},\mathbf{3},-) + (\mathbf{1},\mathbf{1},-) \tag{4.13}$$

in terms of ($\mathrm{SU}(3)_c$, $\mathrm{SU}(3)_L$, PP'). This means that the eigenvalues of $D_y(A_5)^2$ for a **20** representation field B are

$$8 \times \frac{n^2}{R^2}\,, \qquad 3 \times \frac{(n \pm a/2)^2}{R^2}\,, \qquad 3 \times \frac{(n \pm a/2 - 1/2)^2}{R^2}\,, \tag{4.14}$$

where eigenfunctions of B are expanded as $B \propto \cos\frac{ny}{R}, \sin\frac{ny}{R}$ $(\cos\frac{(n+1/2)y}{R}, \sin\frac{(n+1/2)y}{R})$ for $(P,P') = (+,+)$ or $(-,-)$ $((+,-)$ or $(-,+))$. Therefore, the contribution from the **20** representation to the effective potential is

$$\begin{aligned}
&\frac{i}{2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{1}{2\pi R}\sum_{n=-\infty}^{\infty}\left[4\ln\left(-p^2+\left(\frac{n}{R}\right)^2\right) + 3\ln\left(-p^2+\left(\frac{n-a/2}{R}\right)^2\right) + \right.\\
&\qquad\qquad\qquad \left. + 3\ln\left(-p^2+\left(\frac{n-a/2-1/2}{R}\right)^2\right)\right],\\
&= \frac{1}{2}C\sum_{n=1}^{\infty}\frac{1}{n^5}\left[3\cos(\pi n a) + 3\cos(\pi n(a-1))\right],
\end{aligned} \tag{4.15}$$

up to a-independent terms, for one degree of freedom of the fermion. This is just the eq. (4.8).

Now, we have the total effective potential,

$$\begin{aligned}
V_{\mathrm{eff}} &= V_{\mathrm{eff}}^{gauge} + V_{\mathrm{eff}}^{q/l}\\
&= 2C\sum_{n=1}^{\infty}\frac{1}{n^5}(1-\cos(2\pi n\beta)) \times\\
&\qquad \times\left[-\cos(2\pi n a) + (9N_g - 2)\cos(\pi n a) + (8N_g - 6)\cos(\pi n(a-1))\right].
\end{aligned} \tag{4.16}$$

As in the previous section, the 1st derivative, $\partial V_{\mathrm{eff}}/\partial a$ has a factor $\sin(\pi n a)$, which means that the stationary points exist at least at $a = 0$ and $a = 1$. The difference of the heights between two points is given by

$$V_{\mathrm{eff}}(a=0) - V_{\mathrm{eff}}(a=1) = 4(N_g+4)C\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}(1-\cos(2\pi(2n-1)\beta))\,. \tag{4.17}$$

This means that the height of $a=0$ is higher than that of $a=1$ even when $N_g=0$. The vacuum $a=1$ induces the Wilson loop,

$$
\begin{aligned}
W_C &= \exp\left(ig\int_0^{2\pi R} dy \frac{1}{gR} a \frac{\lambda}{2}\right) \\
&= \exp\left(ig\frac{1}{gR}\frac{\lambda}{2}2\pi R\right) = \begin{pmatrix} -1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & -1 \end{pmatrix},
\end{aligned} \tag{4.18}
$$

which suggests $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is broken to $\mathrm{U}(1)_{\mathrm{EM}} \times \mathrm{U}(1)$. This case can not reproduce the correct weak scale VEV as in the previous section.

Then, let us introduce extra fields in the bulk for the suitable electro-weak symmetry breaking. We introduce the extra bulk fields of $N_f^{(\pm)}$ and $N_a^{(\pm)}$ species of hypermultiplets of fundamental and adjoint representations, respectively. The extra matter contributions for the effective potential are given by [12, 13]

$$
\begin{aligned}
V_{\mathrm{eff}}^{\mathrm{extra-m}} = 2C\sum_{n=1}^{\infty}\frac{1}{n^5}(1-\cos(2\pi n\beta))\Big[& N_a^{(+)}\cos(2\pi n a) + N_a^{(-)}\cos(\pi n(2a-1)) + \\
& + \left(2N_a^{(+)} + 6N_a^{(-)} + N_f^{(+)}\right)\cos(\pi n a) + \\
& + \left(6N_a^{(+)} + 2N_a^{(-)} + N_f^{(-)}\right)\cos(\pi n(a-1))\Big].
\end{aligned} \tag{4.19}
$$

There are three phases as eq. (3.18). As in the previous section, the phase structure is obtained by the bulk field content, which is calculated by the 2nd derivatives at $a=0$ and $a=1$,

$$
\begin{aligned}
\left.\frac{\partial^2 V_{\mathrm{eff}}}{\partial a^2}\right|_{a=0} \simeq 2C\pi^2(2\pi\beta)^2\Big[& \left(9N_g - 6 + 6N_a^{(+)} + 6N_a^{(-)} + N_f^{(+)}\right)\left(-\frac{1}{2}\ln(2\pi\beta) + \frac{3}{4}\right) + \\
& + \left(-8N_g + 6 - 6N_a^{(+)} - 6N_a^{(-)} - N_f^{(-)}\right)\frac{1}{2}\ln 2\Big],
\end{aligned} \tag{4.20}
$$

$$
\begin{aligned}
\left.\frac{\partial^2 V_{\mathrm{eff}}}{\partial a^2}\right|_{a=1} \simeq 2C\pi^2(2\pi\beta)^2\Big[& \left(8N_g - 10 + 10N_a^{(+)} + 2N_a^{(-)} + N_f^{(-)}\right)\left(-\frac{1}{2}\ln(2\pi\beta) + \frac{3}{4}\right) + \\
& + \left(-9N_g + 2 - 2N_a^{(+)} - 10N_a^{(-)} - N_f^{(+)}\right)\frac{1}{2}\ln 2\Big].
\end{aligned} \tag{4.21}
$$

Here we use eqs. (3.19) and (3.20). As mentioned in the previous section, $a=0$ point becomes the global minimum when $\frac{\partial^2 V_{\mathrm{eff}}}{\partial a^2}|_{a=0} > 0$ and $V_{\mathrm{eff}}|_{a=0} < V_{\mathrm{eff}}|_{a=1}$. On the other hand, a small VEV, $(0<)a \ll 1$ is realized when $\frac{\partial^2 V_{\mathrm{eff}}}{\partial a^2}|_{a=0} \lesssim 0$ and $\frac{\partial^2 V_{\mathrm{eff}}}{\partial a^2}|_{a=1} < 0$.

Let us consider the situation all three generation quarks and leptons exist in the bulk. For both cases at least $\mathcal{O}(10)$ numbers of $N_f^{(-)}$ are required from eq. (4.20). The vacuum, $a=0$, is realized, for instance, when the extra bulk fields are introduced as $N_a^{(+)}=0$,

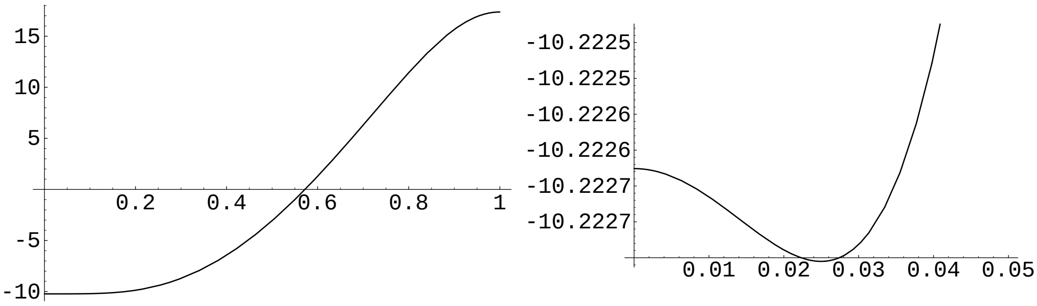

Figure 2: The effective potential in the case of $N_a^{(+)} = N_f^{(+)} = 0$, $N_a^{(-)} = 0$, $N_f^{(-)} = 42$ with $\beta = 0.1$ and $N_g = 3$. The unit is $C = 3/64\pi^7 R^5$. The horizontal line shows $0 \leq a \leq 1$ and $0 \leq a \leq 0.05$.

$N_f^{(+)} = 0$, $N_a^{(-)} = 0$, and $N_f^{(-)} = 50$. While in the TeV scale compactification, the global minimum $a \ll 1$ is possible when, for examples, $N_a^{(+)} = 0$, $N_f^{(+)} = 0$, $N_a^{(-)} = 0$, $N_f^{(-)} = 42$. Figure 2 shows the V_{eff} in the region of $0 \leq a \leq 1$ and $0 \leq a \leq 0.05$. The minimum exists at $a = 0.025$, which is around the suitable magnitude of the weak scale in the TeV scale compactification. The mass squared of the "Higgs field" is given by

$$m_{A_5}^2 \sim \left(\frac{0.012 g_4}{R}\right)^2 \sim (120 g_4^2 \, \text{GeV})^2 \,, \tag{4.22}$$

where $g_4 = \mathcal{O}(1)$ and $\beta = 0.1$.

As in the previous section, the suitable vacuum can be obtained by less numbers of extra bulk fields when only the 2nd and 3rd generations, or only the 3rd generation quarks and leptons exist in the bulk. As for the extra residual U(1) gauge symmetry, we assume it is broken by an extra elementally Higgs field.

5. Summary and discussion

We have studied the possibility of the dynamical symmetry breaking in the gauge-Higgs unification in the 5D $\mathcal{N} = 1$ SUSY theory compactified on an orbifold, $S^1/\mathbb{Z}_2$. We have considered $\mathrm{SU}(3)_c \times \mathrm{SU}(3)_W$ and SU(6) models, where the gauge symmetries are reduced to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)$, respectively, through the orbifolding boundary conditions. Our setup is quarks and leptons are bulk fields, so that Yukawa interactions can be derived from the 5D gauge interactions. We have calculated the one loop effective potential of "Higgs doublets" and analyzed the vacuum structure of the models. We found that the effects of bulk quarks and leptons destabilize the suitable electro-weak symmetry breaking vacuum. We showed that the introduction of suitable numbers of extra bulk fields possessing the suitable representations makes two appropriate scenarios be possible. One is the situation that the one loop effective potential chooses symmetric vacuum at the compactification scale, and the electro-weak symmetry breaking is occurred by other effects in the low energy. The other is the situation that the one loop effective potential chooses the suitable electro-weak vacuum in a few TeV compactification, where the masses of "Higgs doublets" are $\mathcal{O}(100)$ GeV. In this case we can obtain the suitable electro-weak symmetry breaking with vanishing tree-level quartic couplings. It is because the one loop effective potentials have the form of cos-function, which is the characteristic feature of the Wilson line phase. And also the introduction of the bulk fields with $\eta\eta' = -$ is crucial for inducing the electro-weak energy scale [12], which is smaller than the compactification scale. The large number of bulk fields, $N_{a,f}^{(-)} = \mathcal{O}(10)$, makes the "Higgs" mass rather large, although there are no tree-level quartic couplings in this model.

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References

[1] N.S. Manton, *A new six-dimensional approach to the Weinberg-Salam model*, *Nucl. Phys.* **B 158** (1979) 141;
D.B. Fairlie, *Two consistent calculations of the Weinberg angle*, *J. Phys.* **G 5** (1979) L55; *Higgs' fields and the determination of the Weinberg angle*, *Phys. Lett.* **B 82** (1979) 97.

[2] Y. Hosotani, *Dynamical mass generation by compact extra dimensions*, *Phys. Lett.* **B 126** (1983) 309; *Dynamics of nonintegrable phases and gauge symmetry breaking*, *Ann. Phys. (NY)* **190** (1989) 233; *Dynamical gauge symmetry breaking as the casimir effect*, *Phys. Lett.* **B 129** (1983) 193; *Dynamical gauge symmetry breaking and left-right asymmetry in higher dimensional theories*, *Phys. Rev.* **D 29** (1984) 731.

[3] N.V. Krasnikov, *Ultraviolet fixed point behavior of the five-dimensional Yang-Mills theory, the gauge hierarchy problem and a possible new dimension at the TeV scale*, *Phys. Lett.* **B 273** (1991) 246;
H. Hatanaka, T. Inami and C.S. Lim, *The gauge hierarchy problem and higher dimensional gauge theories*, *Mod. Phys. Lett.* **A 13** (1998) 2601 [hep-th/9805067];
G.R. Dvali, S. Randjbar-Daemi and R. Tabbash, *The origin of spontaneous symmetry breaking in theories with large extra dimensions*, *Phys. Rev.* **D 65** (2002) 064021 [hep-ph/0102307];
N. Arkani-Hamed, A.G. Cohen and H. Georgi, *Electroweak symmetry breaking from dimensional deconstruction*, *Phys. Lett.* **B 513** (2001) 232 [hep-ph/0105239];
I. Antoniadis, K. Benakli and M. Quiros, *Finite Higgs mass without supersymmetry*, *New J. Phys.* **3** (2001) 20 [hep-th/0108005].

[4] M. Kubo, C.S. Lim and H. Yamashita, *The hosotani mechanism in bulk gauge theories with an orbifold extra space $S_1/\mathbb{Z}_2$*, *Mod. Phys. Lett.* **A 17** (2002) 2249 [hep-ph/0111327].

[5] L.J. Hall, Y. Nomura and D.R. Smith, *Gauge-higgs unification in higher dimensions*, *Nucl. Phys.* **B 639** (2002) 307 [hep-ph/0107331].

[6] G. Burdman and Y. Nomura, *Unification of Higgs and gauge fields in five dimensions*, *Nucl. Phys.* **B 656** (2003) 3 [hep-ph/0210257].

[7] N. Haba and Y. Shimizu, *Gauge-Higgs unification in the 5 dimensional E_6, E_7 and E_8 guts on orbifold*, *Phys. Rev.* **D 67** (2003) 095001 [hep-ph/0212166].

[8] C.A. Scrucca, M. Serone and L. Silvestrini, *Electroweak symmetry breaking and fermion masses from extra dimensions*, *Nucl. Phys.* **B 669** (2003) 128 [hep-ph/0304220].

[9] I. Gogoladze, Y. Mimura and S. Nandi, *Gauge Higgs unification on the left-right model*, *Phys. Lett.* **B 560** (2003) 204 [hep-ph/0301014]; *Unification of gauge, Higgs and matter in extra dimensions*, *Phys. Lett.* **B 562** (2003) 307 [hep-ph/0302176];
I. Gogoladze, Y. Mimura, S. Nandi and K. Tobe, *Test of gauge-Yukawa unification*, *Phys. Lett.* **B 575** (2003) 66 [hep-ph/0307397].

[10] C. Csáki, C. Grojean, H. Murayama, L. Pilo and J. Terning, *Gauge theories on an interval: unitarity without a Higgs*, *Phys. Rev.* **D 69** (2004) 055006 [hep-ph/0305237].

[11] K.W. Choi et al., *Electroweak symmetry breaking in supersymmetric gauge-Higgs unification models*, *J. High Energy Phys.* **02** (2004) 037 [hep-ph/0312178].

[12] N. Haba, Y. Hosotani, Y. Kawamura and T. Yamashita, *Dynamical symmetry breaking in gauge-Higgs unification on orbifold*, hep-ph/0401183.

[13] N. Haba and T. Yamashita, *The general formula of the effective potential in 5D SU(N) gauge theory on orbifold*, *J. High Energy Phys.* **02** (2004) 059 [hep-ph/0401185].

[14] N. Haba, M. Harada, Y. Hosotani, and Y. Kawamura, *Dynamical rearrangement of gauge symmetry on the orbifold $S_1/\mathbb{Z}_2$ Nucl. Phys.* **B 657** (2003) 169 [hep-ph/0212035].

[15] N. Haba, Y. Hosotani and Y. Kawamura, *Classification and dynamics of equivalence classes in SU(N) gauge theory on the orbifold $S_1/\mathbb{Z}_2$*, *Prog. Theor. Phys.* **111** (2004) 265 [hep-ph/0309088].

[16] N. Arkani-Hamed, T. Gregoire and J. Wacker, *Higher dimensional supersymmetry in 4D superspace*, *J. High Energy Phys.* **03** (2002) 055 [hep-th/0101233].

[17] D.E. Kaplan and T.M.P. Tait, *New tools for fermion masses from extra dimensions*, *J. High Energy Phys.* **11** (2001) 051 [hep-ph/0110126].

[18] J. Scherk and J.H. Schwarz, *Spontaneous breaking of supersymmetry through dimensional reduction*, *Phys. Lett.* **B 82** (1979) 60; *How to get masses from extra dimensions*, *Nucl. Phys.* **B 153** (1979) 61.

[19] P. Fayet, *Supersymmetric grand unification in a six-dimensional space-time*, *Phys. Lett.* **B 159** (1985) 121; *Six-dimensional supersymmetric QED, R invariance and $N = 2$ supersymmetry breaking by dimensional reduction*, *Nucl. Phys.* **B 263** (1986) 649.

[20] I. Antoniadis, *A possible new dimension at a few TeV*, *Phys. Lett.* **B 246** (1990) 377;
I. Antoniadis, C. Muñoz and M. Quiros, *Dynamical supersymmetry breaking with a large internal dimension*, *Nucl. Phys.* **B 397** (1993) 515 [hep-ph/9211309];
A. Pomarol and M. Quiros, *The standard model from extra dimensions*, *Phys. Lett.* **B 438** (1998) 255 [hep-ph/9806263];
I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, *Soft masses in theories with supersymmetry breaking by TeV-compactification*, *Nucl. Phys.* **B 544** (1999) 503 [hep-ph/9810410];

A. Delgado, A. Pomarol and M. Quiros, *Supersymmetry and electroweak breaking from extra dimensions at the TeV-scale*, *Phys. Rev.* **D 60** (1999) 095008 [hep-ph/9812489].

[21] G. von Gersdorff, M. Quiros and A. Riotto, *Radiative Scherk-Schwarz supersymmetry breaking*, *Nucl. Phys.* **B 634** (2002) 90 [hep-th/0204041];
G. von Gersdorff and M. Quiros, *Supersymmetry breaking on orbifolds from wilson lines*, *Phys. Rev.* **D 65** (2002) 064016 [hep-th/0110132].

[22] K. Takenaga, *Effect of bare mass on the Hosotani mechanism*, *Phys. Lett.* **B 570** (2003) 244 [hep-th/0305251].

[23] K.R. Dienes, E. Dudas and T. Gherghetta, *Extra spacetime dimensions and unification*, *Phys. Lett.* **B 436** (1998) 55 [hep-ph/9803466]; *Grand unification at intermediate mass scales through extra dimensions*, *Nucl. Phys.* **B 537** (1999) 47 [hep-ph/9806292].

[24] N. Arkani-Hamed, A.G. Cohen and H. Georgi, *Accelerated unification*, hep-th/0108089.

JHEP04(2004)016